Review Big Oh

Sunday, February 11, 2018 10:48 PM

Easy problem

Find and prove a big O bound for the following recurrence relations $j \rightarrow k = \log_j(n)$

$S(n) = 2S(n/2) + n$ Assuming $S(k) = c$ if $k \leq 1$

$S(n/2) = 2S(n/4) + n/2$

$S(n) = 2(2S(n/4) + n/2) + n$

$S(n) = 4S(n/4) + n + n$

$S(n/4) = 2S(n/8) + n/4$

$S(n) = 4(2S(n/8) + n/4) + n + n$

$S(n) = 8S(n/8) + 3n$

let $k = \lceil \log_2(n) \rceil$

$2^k = n$

$S(n) = 2^i S(n/2^i) + i(n)$

$= 2^k S(n/2^k) + kn = 2^k c + kn = nc + n\log_2(n)$

$= n\log_2(n)$

$\ldots) = c\,S(/2^k) + kn = c \cdot c + kn = nc + n\log_2(n)$

$= n\log_2(n)$

$S(n) = 2\,S(n/2) + n^2$

$S(n/2) = 2S(n/4) + \left(\frac{n}{2}\right)^2$

$S(n) = 2\left(2\,S(n/4) + \left(n/2\right)^2\right) + n^2$

$= 4\,S(n/4) + \frac{n^2}{2} + n^2$

$S(n/4) = \underbrace{2\,S(n/8) + \left(\frac{n}{4}\right)^2}$

$S(n) = 4\left(2S(n/8) + \left(\frac{n}{4}\right)^2\right) + \frac{n^2}{2} + n^2$

$= 8\,S(n/8) + \frac{n^2}{4} + \frac{n^2}{2} + n^2$

$= 2^i S(n/2^i) + \sum_{j=0}^{i-1} \frac{n^2}{2^j} = 2^i S(n/2^i) + n^2 \sum_{j=0}^{i} \frac{1}{2^j}$

$2^k S(1) + n^2 \sum_{j=0}^{k-1} \frac{1}{2^j} \leq n \cdot c + n^2(2)$

$O(n^2)$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$\frac{1-\frac{1}{2^i}}{1-\frac{1}{2}} \leq \frac{1}{\frac{1}{2}} = 2$

Prove $2^n \in O(n!)$

$\exists\, c, x_0 \quad \forall x > x_0 \quad f(x) \leq c\, g(x)$

$\exists c, x_0$ $\forall x > x_0$ $f(x) \le c\,g(x)$

$C = 10$
$x_0 = 3$

base case $x = 4$

$2^4 = 16$ $\quad$ $4! = 24$
$10 \cdot 4! =$
$16 < 240$

Assume this is true for some integer $n \ge 4$

we want to show $\underline{2^{n+1}} \le c((n+1)!)$

2^{n+1}
$2 \cdot 2^n$

By induction hypothesis

$\le 2 \cdot c(n!)$ $\quad$ $2 \le n+1$ and $n!$ should be positive

$\le (n+1)c(n!)$
$= c((n+1)!)$ ✓

$\blacksquare$